



CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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9231/23

May/June 2022

2 hours

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1 Find the roots of the equation $z^3 = 7\sqrt{3} - 7i$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi \leq \theta < \pi$. [5]

[illegible]

- 2 (a) Find the coefficient of x^2 in the Maclaurin's series for $-\ln \cos x$. [4]

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- (b) Find the length of the arc of the curve with equation $y = -\ln \cos x$ from the point where $x = 0$ to the point where $x = \frac{1}{4}\pi$. [4]

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3 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -9 & 5 \\ 5 & -8 & 5 \\ 1 & -1 & 2 \end{pmatrix}.$$

(a) Find the eigenvalues of $\mathbf{A}$.

[4]

[illegible]

- (b)** Use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{A}^{-1} = p\mathbf{A}^2 + q\mathbf{I}$, where p and q are constants to be determined. [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

4 It is given that

$$x = -t + \tan^{-1} t \quad \text{and} \quad y = t + \sinh^{-1} t.$$

(a) Show that $\frac{dy}{dx} = -\frac{t^2 + 1 + \sqrt{t^2 + 1}}{t^2}$. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b)** Find the value of $\frac{d^2y}{dx^2}$ when $t = \frac{3}{4}$. [5]

[illegible]

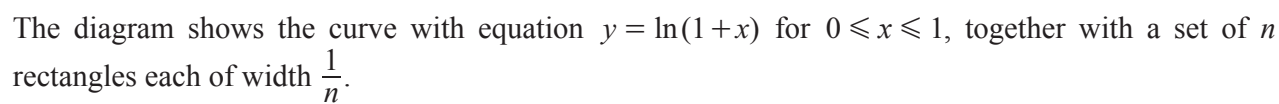
5 Find the solution of the differential equation

$$x(x+7)\frac{dy}{dx} + 7y = x$$

for which $y = 7$ when $x = 1$. Give your answer in the form $y = f(x)$.

[9]

[illegible]



- $$U_n = \frac{1}{n} \ln \frac{(2n)!}{n!} - \ln n. \quad [4]$$

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- (b)** Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 \ln(1+x) dx$. [4]

[illegible]

- (c) By simplifying $U_n - L_n$, show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]

[illegible]

7 The variables x and y are related by the differential equation

$$4 \frac{d^2 y}{dx^2} - y = 3.$$

It is given that, when $x = 0$, $y = -3$ and $\frac{dy}{dx} = 2$.

(a) Find y in terms of x .

[8]

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- (b) Deduce the exact value of x for which $y = 0$. Give your answer in logarithmic form. [3]

8 (a) Find $\int \sin \theta \cos^n \theta d\theta$, where $n \neq -1$. [2]

Let $I_{m,n} = \int_0^{\frac{1}{2}\pi} \sin^m \theta \cos^n \theta \, d\theta$.

(b) Show that, for $m \geq 2$ and $n \geq 0$,

$$I_{m,n} = \frac{m-1}{m+n} I_{m-2,n}. \quad [5]$$

- (c) By considering the binomial expansion of $\left(z + \frac{1}{z}\right)^5$, where $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to show that

$$\cos^5 \theta = a \cos 5\theta + b \cos 3\theta + c \cos \theta,$$

where a , b and c are constants to be determined.

[5]

[illegible]

[4]

[illegible]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

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